Online Appendix to "Private Monitoring and Communication in Cartels: Explaining Recent Collusive Practices"

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1 Static Nash Equilibrium

Assume that $\psi_i\left(q;m,\underline{p}\right)$ depends only on pair-wise price differences; this property can be derived from a consumer choice model with quasi-linear preferences in money. It implies that for all prices:

$$\psi_i\left(q;m,\underline{p}\right) = \psi_i\left(q;m,\underline{p}+\Delta 1_n\right) \quad (1)$$

where 1_n is a vector of ones of length n. Also, assume that the FOC is sufficient to characterize a unique equilibrium for any symmetric cost structure. Let $\psi_i\left(m,\underline{p}\right)$ denote the corresponding vector specifying the probabilities over different quantities $q_i \in \{0,...\overline{m}\}$ for player i.

Lemma 1 *The equilibrium path-through of an increase in marginal costs to prices is 100%. That is, if all marginal costs increase by* Δ*, then the Nash equilibrium prices increase by* Δ*.*

Proof. Firm i's best response problem to set price $p_i^N(c_i)$ to other firms setting prices p_{-i}^N is:

$$p_i^N(c_i) \in \arg\max_{p_i} \sum_{m=\underline{m}}^{\overline{m}} \rho(m)(p_i - c_i)\left(\underline{q_i} \cdot \psi_i\left(m, p_1^N, \ldots, p_i, \ldots, p_m^N\right)\right).$$

where $\underline{q_i}$ is a vector $(0, 1, ...\overline{m})$ and "$\cdot$" is a dot product. The FOC of this problem is

$$\sum_{m=\underline{m}}^{\overline{m}} \rho(m)\left[\left(\underline{q_i} \cdot \psi_i\left(m, \underline{p^N}\right) + (p_i - c_i)\,\underline{q_i} \cdot \frac{\partial \psi_i\left(m, \underline{p^N}\right)}{\partial p_i}\right)\right] = 0$$

Suppose it is satisfied at $\underline{p^N}(c)$. Consider costs $c + \Delta$ and evaluate the FOC at prices $\underline{p^N}(c) + \Delta 1_n$:

$$\sum_{m=\underline{m}}^{\overline{m}} \rho(m)\left[\left(\underline{q_i} \cdot \psi_i\left(m, \underline{p^N} + \Delta 1_n\right) + ((p_i + \Delta) - (c_i + \Delta))\,\underline{q_i} \cdot \frac{\partial \psi_i\left(m, \underline{p^N} + \Delta 1_n\right)}{\partial p_i}\right)\right]$$
$$= \sum_{m=\underline{m}}^{\overline{m}} \rho(m)\left[\left(\underline{q_i} \cdot \psi_i\left(m, \underline{p^N}\right) + (p_i - c_i)\,\underline{q_i} \cdot \frac{\partial \psi_i\left(m, \underline{p^N}\right)}{\partial p_i}\right)\right] = 0$$

where we used (1) to simplify. Hence, the FOC holds at these prices. ■

100% pass-through implies that $p^N(c) = c + const$. Finally, note that the above lemma holds even if costs are asymmetric: when firms have a vector of marginal costs $\underline{c}$ then $\underline{p}^N(\underline{c} + \Delta 1_n) = \underline{p}^N(\underline{c}) + \Delta 1_n$.

2 Collusive Scheme without Condition (1)

Suppose there are two firms and $m \in \{\underline{m}, ..., \overline{m}\}$ with $\underline{m} > 0$ and even. Moreover, $\rho(\underline{m}) = 1 - \varepsilon$ and $\rho(m) = \hat{\rho}(m)\varepsilon$ for $m > \underline{m}$, where $\hat{\rho}(m)$ is a probability distribution over m conditional on $m > \underline{m}$, which is positive for all $\underline{m} < m \leq \overline{m}$. We allow $\overline{m}$ to be high enough so that condition (1) from the paper is violated. In words, the belief is that the total demand is $m = \underline{m}$ with a very high probability, but there is a right tail of high demand realizations which could be quite long.

Consider the following collusive scheme: at the end of each period, both firms report simultaneously their sales, (r_1, r_2). If the sum of reports is less than $\underline{m}$, then firms move to a punishment phase. (Note the punishment does not then happen on the equilibrium path). If the sum of reports is at least $\underline{m}$, then if a firm sold less than $\underline{m}/2$, it receives a payment z from the other firm for every unit below $\underline{m}/2$. In other words, each firm is promised to sell at least $\underline{m}/2$ units and for all sales below $\underline{m}/2$ it is compensated with a transfer z from the other firm.

To keep the formulas simple, we describe the pricing stage of the scheme for $\underline{m} = 100$ (so that each firm is promised to sell at least 50 units), but it should be clear that the scheme works for a general $\underline{m}$. With such a scheme, in the price-setting stage firms maximize:

$$\max_{p_i} \rho(100) [\sum_{q_i=0}^{100} ((p_i - c) q_i + (50 - q_i) z) \psi_i (q_i, \underline{m}, \underline{p})] + \sum_{m=101}^{\overline{m}} \rho(m) \begin{bmatrix} \sum_{q_i=0}^{50} ((p_i - c) q_i + (50 - q_i) z) \psi_i (q_i, m, \underline{p}) \\ + \sum_{q_i=51}^{m-50} ((p_i - c) q_i) \psi_i (q_i, m, \underline{p}) \\ + \sum_{q_i=m-49}^{m} ((p_i - c) q_i + (m - 50 - q_i) z) \psi_i (q_i, m, \underline{p}) \end{bmatrix}$$

where in the second summation, the first term corresponds to firm i being below its quota, the second term to both firms meeting their quota, and the last term to firm $-i$ being below its quota. This objective can be re-written as:

$$\max_{p_i} \rho(100) [\sum_{q_i=0}^{100} ((p_i - (c+z)) q_i) \psi_i (q_i, 100, \underline{p}) + 50z] + \sum_{m=101}^{\overline{m}} \rho(m) \begin{bmatrix} \sum_{q_i=0}^{50} ((p_i - (c+z)) q_i + 50z) \psi_i (q_i, m, \underline{p}) \\ + \sum_{q_i=51}^{m-50} ((p_i - c) q_i) \psi_i (q_i, m, \underline{p}) \\ + \sum_{q_i=m-49}^{m} ((p_i - (c+z)) q_i + (m - 50) z) \psi_i (q_i, m, \underline{p}) \end{bmatrix}$$

which means that conditional on $m = 100$ the profit is as if marginal cost was $c + z$ rather than c, while the profit expression is more complicated when $m > 100$. Yet, as in the case of the lysine strategy, in the price-setting stage we can analyze prices using this modified static game. Suppose that for every z this static game has a unique Nash equilibrium, that is continuous in ε and z. Let $p^N(z, \varepsilon)$ be that price. By continuity, for small ε, $p^N(z, \varepsilon)$ is close to $p^N(z, 0)$. Assume that $p^N(z, 0)$ is strictly and increasing in z without bound (this is our assumption A4). [1] Under these assumptions on $p^N(z, \varepsilon)$, for any price $\hat{p}$ it is possible to find $\bar{\varepsilon} > 0$ small enough such that for all $\varepsilon \leq \bar{\varepsilon}$ we can find a z so that in the pricing stage the firms would set a symmetric price of $\hat{p}$.

Now consider the reporting stage. If a firm sells q_i units it can affect its transfer only by reporting $r_i < \underline{m}/2$, since all reports above $\underline{m}/2$ do not affect how much it pays (in that case payment depends on the report of the other firm only).

To show that truthful reporting is incentive compatible, we need to make one more assumption. For any q_i denote by $\tilde{R}(q_i)$ the set of the potentially profitable reports, that is, report $r_i \in \tilde{R}(q_i)$ if it is strictly less than q_i and strictly less than $\underline{m}/2$ ($r_i < 50$ in our example). We assume that for every price p_i (including off-equilibrium prices) and any q_i, firm i assigns positive probability to the other firm selling q_j such that $r_i + q_j < \underline{m}$ for any $r_i \in \tilde{R}(q_i)$. Moreover, we assume that this belief is uniformly bounded away from zero by an amount A. In words, we assume that a firm reporting less than their quota and less than their actual sales, assigns a positive probability that the other firm sold so few units that this will make the sum of reports less than $\underline{m}$, which clearly indicates a deviation. A sufficient condition for this assumption to hold is: a) if a firm sells $q_i < \underline{m}$, it assigns a positive probability to $m = \underline{m}$; b) if a

[1] Alternatively, one could assume that $p^N(z, \varepsilon)$ is increasing in z for any ε, but that is more difficult to verify than $p^N(z, 0)$ is increasing since $p^N(z, 0)$ corresponds to a static Nash equilbrium of a much simpler game. Also, while it is natural to expect that $p^N(z, \varepsilon)$ is increasing in z for a small ε (because the main effect of z on the static game is analogous to changes in c), for a large ε the impact of z is complex and we do not know how restrictive that assumption would be.

firm sells $q_i > \underline{m}$, it assigns a positive probability to the other firm selling less than $\frac{\underline{m}}{2}$.

Next, suppose that if $r_1 + r_2 < \underline{m}$ then the collusive mechanism will trigger a mutual destruction of value X which is at least:

$$\frac{50z}{A} < X$$

With that punishment, firm i will not find it profitable to report $r_i < q_i$ because the gain is bounded from above by $50z$ and the loss is bounded from below by AX. The same punishment can be used to deter firms from reneging on payments.

In the dynamic game, we can implement the punishment by a threat of infinite reversion to a stage game Nash equilibrium. Since the collusive scheme increases prices from $p^N(0, \varepsilon)$ to $\hat{p}$, the Nash reversion corresponds to:

$$X = \frac{\delta}{1-\delta}\left(\hat{p} - p^N(0, \varepsilon)\right)\frac{\mu}{2}$$

Hence for any $\hat{p} > p^N(0, \varepsilon)$ (and $\varepsilon \leq \bar{\varepsilon}$), for δ sufficiently close to one, no firm would find it profitable to under-report sales.

This construction gives us a semi-public perfect collusive equilibrium. Firms start in a collusive state and are recommended to set prices $(\hat{p}, \hat{p})$. After every period they simultaneously report their sales. If the reports add up to at least $\underline{m}$ then any firm with sales below $\frac{\underline{m}}{2}$ is compensated by the other firm with a transfer of z per unit of shortfall. If payments are made, we move to the next period. If in any period reports do not add up to at least $\underline{m}$ or a firm reneges on payments, we switch to the punishment state in which we play the static Nash equilibrium forever.

Note that this scheme does not respond to firms over-reporting sales because these IC reporting incentives are slack. If a firm sells $q_i \geq 50$ then over-reporting does not change the payoffs at all. If a firm sells $q_i < 50$ then over-reporting reduces the transfer received without any compensating benefit because on the path there are no punishments for low aggregate sales reports, which is different from the lysine strategy.

This scheme has the desirable property that it allows firms to collude using only balanced transfers and value burning that occurs only as an off-equilibrium threat (yet, a credible one), a stark contrast with the lysine strategy we described. It is an open question for which demand structures it is possible to construct collusive equilibria that do not use value burning on the equilibrium path. The example in the next section illustrates for some demand structures that it is a part of the optimal mechanism (and in case $m \in \{0, 1\}$ one can show that it is necessary for any collusive scheme). These examples suggest a conjecture that value burning is necessary if (and possibly only if) $m = 0$ is assigned a positive probability. Finally, the considerations of value burning and incentives for over-reporting it can trigger, suggest that having long right tail in the distribution of m is much easier to handle for the cartel than having a long left tail.

3 Optimal Mechanism for the Two Unit Demand Case

3.1 Model

In this section we study a model with two firms, $m \in \{0, 1, 2\}$ and a simple demand structure. We first consider a static model in which firms can design a mechanism specifying transfers and value destruction as a function of reported sales. The payoffs from that mechanism imply an upper bound on equilibrium payoffs in any semi-public equilibrium in which players report sales without delay. We then show that if players are sufficiently patient that upper bound is achievable. Interestingly, the optimal equilibria mimic the lysine strategy that we constructed for the general case. Although the lysine strategy is unlikely to be optimal in general, we find it informative to see that the same two main instruments are used in the optimal equilibria as in our lysine strategy: transfers from players with higher sales to players with low sales and the threat of destruction of value in case total reports are low.

Taking a mechanism design approach, we focus on a highly simplified duopoly case.[2] Nature chooses the number of active buyers from the set $\{0, 1, 2\}$ where $\rho_m \equiv \rho(m)$ is the probability that demand is $m \in \{0, 1, 2\}$. Each active buyer buys one unit of output. If an active buyer is offered prices p_1 and p_2 then she buys from firm 1 with probability $\xi(p_2 - p_1)$ and from firm 2 with probability $1 - \xi(p_2 - p_1)$; thus, the probability of purchase decisions depends only on the price difference. Buyers' purchase decisions are independent.

It is assumed ξ is a differentiable, increasing function and $\xi(0) = 1/2$. Furthermore, we assume that ξ is such that, given the scheme we construct, the FOC of the price-setting problem is sufficient for optimality.[3] Using ξ and the independence of consumer choices we get:

$$
\begin{aligned}
\psi(1; 1, p_1, p_2) &= \xi(p_2 - p_1) \\
\psi(2; 2, p_1, p_2) &= \xi(p_2 - p_1)^2 \\
\psi(1; 2, p_1, p_2) &= 2\xi(p_2 - p_1)\left[1 - \xi(p_2 - p_1)\right]. \\
\psi(0; 2, p_1, p_2) &= \left[1 - \xi(p_2 - p_1)\right]^2
\end{aligned}
$$

Reports are restricted so that $r_i \in \{0, 1, 2\}$. $\eta_{i,j}(p_1, p_2)$ will denote the probability that $q_2 = j$ given $q_1 = i$ and firms' prices, and $\psi(q_1; m, p_1, p_2)$ is the probability of firm 1 having sales of q_1 given total demand is m and given firms' prices.

3.2 Characterization of an Optimal Mechanism

A collusive mechanism consists of a recommended price pair (p_1, p_2) and a transfer rule that depends on reported sales. A transfer rule $\{t_1(r_1, r_2),\ t_2(r_2, r_1)\}$ specifies net transfers received by the two players conditional on the reports. The mechanism

[2] With some additional notation, we believe results can be extended in a straightforward manner to when there are n firms.

[3] See footnote 5 for discussion of sufficient conditions.

is feasible if

$$t_1(r_1, r_2) + t_2(r_2, r_1) \leq 0, \forall (r_1, r_2). \tag{2}$$

Moreover, we restrict the transfers to be bounded:

$$t_1(r_1, r_2), t_2(r_2, r_1) \in [-x, x], \ \forall (r_1, r_2), \tag{3}$$

where $x > 0$. Restriction (3) may be needed for the existence of an optimal mechanism because total demand is inelastic. If bigger inter-firm transfers are more likely to trigger an investigation by the antitrust authorities, cartel members may want to put a bound on those transfers. Thus, the technical assumption in (3) may have an economic rationale as well.[4] The mechanism is incentive compatible if both firms find it optimal to set the recommended prices and report their realized sales truthfully (incentive compatibility requires truthful reporting if a firm follows the price recommendation but not otherwise).

A collusive mechanism is symmetric if $t_1(r_1, r_2) = t_2(r_2, r_1) = t(r_1, r_2)$ and $p_1 = p_2 = \widehat{p}$. Our goal is to describe an optimal symmetric incentive compatible feasible collusive mechanism.

Anticipating that both firms will truthfully report their sales, firm 1's expected payoff at the price stage is:

$$\begin{aligned} &\rho_0 t(0,0) + \rho_1 [\psi(1;1)(p_1 - c + t(1,0)) + (1 - \psi(1;1)) t(0,1)] \\ &+\rho_2[\psi(2;2)(2(p_1 - c) + t(2,0)) + \psi(1;2)(p_1 - c + t(1,1)) + (\psi(0;2)) t(0,2)], \end{aligned} \tag{4}$$

where we have suppressed the dependence of $\psi(\cdot)$ on firms' prices. The FOC for price

[4] When we use the optimal mechanism to construct equilibria in the repeated game, a natural constraint on x comes from the incentive constraints that players may renege on payments.

(assuming truthful reporting) gives us the symmetric equilibrium price $\widehat{p}$:[5]

$$\begin{aligned} 0 &= \rho_1 \left\{ \frac{\partial \psi(1;1)}{\partial p_1} \left[\widehat{p} - c + t(1,0) - t(0,1) \right] + \frac{1}{2} \right\} \\ &\quad + \rho_2 \left\{ \frac{\partial \psi(2;2)}{\partial p_1} \left[2(\widehat{p} - c) + t(2,0) \right] + 2\psi(2;2) \right. \\ &\quad \left. + \frac{\partial \psi(1;2)}{\partial p_1} \left[\widehat{p} - c + t(1,1) \right] + \psi(1;2) + \frac{\partial (\psi(0;2))}{\partial p_1} t(0,2) \right\}. \end{aligned} \tag{5}$$

Using our assumptions on ψ, this can be simplified to

$$\begin{aligned} 0 &= \rho_1 \left\{ -\xi'(0) \left[\widehat{p} - c + t(1,0) - t(0,1) \right] + \frac{1}{2} \right\} \\ &\quad + \rho_2 \left\{ -\xi'(0) \left[2(\widehat{p} - c) + t(2,0) - t(0,2) \right] + 1 \right\}. \end{aligned} \tag{6}$$

Solving it, the symmetric equilibrium price is:

$$\widehat{p} = c + \frac{1}{2\xi'(0)} + \frac{\rho_1}{\rho_1 + 2\rho_2} \left[t(0,1) - t(1,0) \right] + \frac{\rho_2}{\rho_1 + 2\rho_2} \left[t(0,2) - t(2,0) \right]. \tag{7}$$

When $t(0,1) - t(1,0)$ is higher, a firm benefits more from being the firm with zero demand when market demand is one. There is then an incentive for a firm to raise price and that is why the equilibrium price is increasing in $t(0,1) - t(1,0)$. A similar logic explains why the equilibrium price is increasing in $t(0,2) - t(2,0)$.

Let us consider the incentive compatibility constraints (ICCs) in the reporting stage. Suppose $q_1 = 2$, in which case firm 1 knows that firm 2 sold zero units. The ICC for truthful reporting is

$$t(2,0) \geq t(1,0), t(0,0). \tag{8}$$

[5] Sufficient conditions for the equilibrium price to be defined by the FOC is that ξ is linear when it achieves values in $(0,1)$ and transfers are not too large. When ξ is linear, it is straightforward to show that the SOC is

$$-2\xi'(p_2 - p_1) \left\{ \rho_1 + 2\rho_2 + \rho_2 \xi'(p_2 - p_1) \left[2t(1,1) - t(2,0) - t(0,2) \right] \right\} < 0.$$

Since $\xi'(p_2 - p_1) > 0$, then the SOC is satisfied as long as

$$t(2,0) + t(0,2) - 2t(1,1) \geq 0,$$

or it is sufficiently close to zero. This expression will equal zero for the optimal mechanism.

When $q_1 = 1$, the ICCs for truthful reporting are

$$\eta_{1,1}(p_1, \widehat{p})\, t(1,1) + \left(1 - \eta_{1,1}(p_1, \widehat{p})\right) t(1,0) \geq \eta_{1,1}(p_1, \widehat{p})\, t(2,1) + \left(1 - \eta_{1,1}(p_1, \widehat{p})\right) t(2,0) \tag{9}$$

$$\eta_{1,1}(p_1, \widehat{p})\, t(1,1) + \left(1 - \eta_{1,1}(p_1, \widehat{p})\right) t(1,0) \geq \eta_{1,1}(p_1, \widehat{p})\, t(0,1) + \left(1 - \eta_{1,1}(p_1, \widehat{p})\right) t(0,0) \tag{10}$$

$\eta_{1,1}(p_1, \widehat{p})$ is the probability that firm 1 assigns to firm 2 selling one unit, given firm 1 sold one unit and the price pair. By (9), firm 1 prefers to report having sold one unit than reporting two units; and by (10), firm 1 prefers to report having sold one unit than reporting zero units. It is necessary for the mechanism to be incentive compatible that (9) and (10) hold at $p_1 = \widehat{p}$. However, that is not sufficient, since the firm may have a profitable "double-deviation"; that is, deviating with price *and* report. In our construction in the proof of Theorem 3, we use only these necessary conditions and then verify that the firm has no incentive to misreport even if it deviates in price as well.

Finally, when $q_1 = 0$, the ICCs are

$$\begin{aligned} & \eta_{0,2}(p_1, \widehat{p})\, t(0,2) + \eta_{0,1}(p_1, \widehat{p})\, t(0,1) \\ & + \left(1 - \eta_{0,2}(p_1, \widehat{p}) - \eta_{0,1}(p_1, \widehat{p})\right) t(0,0) \\ \geq\; & \eta_{0,2}(p_1, \widehat{p})\, t(1,2) + \eta_{0,1}(p_1, \widehat{p})\, t(1,1) \\ & + \left(1 - \eta_{0,2}(p_1, \widehat{p}) - \eta_{0,1}(p_1, \widehat{p})\right) t(1,0) \end{aligned} \tag{11}$$

$$\begin{aligned} & \eta_{0,2}(p_1, \widehat{p})\, t(0,2) + \eta_{0,1}(p_1, \widehat{p})\, t(0,1) \\ & + \left(1 - \eta_{0,2}(p_1, \widehat{p}) - \eta_{0,1}(p_1, \widehat{p})\right) t(0,0) \\ \geq\; & \eta_{0,2}(p_1, \widehat{p})\, t(2,2) + \eta_{0,1}(p_1, \widehat{p})\, t(2,1) \\ & + \left(1 - \eta_{0,2}(p_1, \widehat{p}) - \eta_{0,1}(p_1, \widehat{p})\right) t(2,0) \end{aligned} \tag{12}$$

(again it is necessary that these hold for $p_1 = \widehat{p}$, and sufficient if they hold for all p_1).

Substituting (7) into the expected payoff in (4), the (relaxed) problem is to choose a transfer function $t(\cdot)$ to maximize

$$\rho_0 t(0,0) + (\rho_1/2)\left[t(1,0) + t(0,1)\right] \tag{13}$$
$$+\left(\frac{1-\rho_0-\rho_1}{4}\right)\left[t(2,0) + 2t(1,1) + t(0,2)\right]$$
$$+\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + \left(\frac{\rho_1}{2}\right)\left[t(0,1) - t(1,0)\right] + \left(\frac{1-\rho_0-\rho_1}{2}\right)\left[t(0,2) - t(2,0)\right]$$

subject to the feasibility constraints (2)-(3) and the ICCs (8)-(12). The proof of Theorem 3 is

provided at the end of the appendix.

Theorem 3: Under the assumptions of Section 5, if the high demand state is most likely ($\rho_2 > \rho_0, \rho_1$), an optimal symmetric mechanism is:

$$\begin{aligned} t(0,0) &= -x \\ t(0,1) &= 0, t(1,0) = -x \\ t(0,2) &= x, t(2,0) = -x \\ t(1,1) &= 0 \\ t(r_1, r_2) &= -x \text{ if } r_1 + r_2 > 2 \end{aligned}$$

and the resulting expected firm payoff is:

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + (\rho_2 - \rho_0)\, x.$$

If the low demand state is most likely ($\rho_0 > \rho_1, \rho_2$), there does not exist any symmetric mechanism yielding payoffs in excess of those produced by a stage game Nash equilibrium.

When the low demand state is most likely, collusion cannot be sustained.[6] We do not have a characterization when the medium demand state is most likely ($\rho_1 >$

[6] As earlier work on private monitoring suggests, delay in exchanging reports will presumably be necessary to support collusion when $\rho_0 > \rho_1, \rho_2$.

ρ_0, ρ_2).[7] When the high demand state is most likely, collusion can be sustained and the optimal mechanism has the following properties. When market demand is two units and one firm sold both of those units, that firm is required to make a transfer of x to the firm that sold nothing. When both firms sold one unit, there are no transfers. When market demand is one unit, the firm having sold that unit incurs a penalty of x and the other firm receives no payment, so value is destroyed. Finally, when market demand is zero, both firms incur a penalty of x, and again there is an inefficiency. The remainder of this section will explore this optimal mechanism; thus, we will be assuming $\rho_2 > \rho_0, \rho_1$.

This mechanism provides an upper bound on collusive equilibrium payoffs in our repeated game (where transfers are not contractible so firms must find it optimal to pay them) for any symmetric semi-public perfect equilibrium with firms reporting without delays. The reason is that for any such Pareto-efficient equilibrium, whatever can be achieved by using continuation payoffs to provide incentives, can be also achieved in our static mechanism with transfers (which is not necessarily true for equilibria with delays in reporting). The complication is that in the repeated game the bound x is endogenous: for a given discount factor, if the collusive scheme calls for firm i to pay too much (either as a transfer to the other firm or as value burning), it will prefer to renege since punishments are bounded by the difference between the best and worst equilibrium payoffs. A different way of bounding x arises if we assume that the demand is inelastic up to some choke price and drops down to zero above that price (as we discussed in Section 2 of the paper, this is a more realistic assumption than the demand being perfectly inelastic for all prices). For $\hat{p}$ in (7) not to exceed the choke price it must be that transfers do not exceed some level, giving us an upper bound on x. Since that bound is independent of δ, it leads to an upper bound on per-period collusive payoffs that is independent of δ. For example, it

[7] When $\rho_1 > \rho_0, \rho_2$, we can characterize an optimal mechanism when a firm deviates in its price *or* in its reports, but a mechanism immune to deviating simultaneously in price and report has thus far alluded us. The difficulty is in verifying that there are no profitable double deviations.

means that if $m = 0$ is the most likely level of market demand, there does not exist a symmetric semi-public perfect equilibrium without delay with payoffs higher than the static Nash payoff.

Our use of a static mechanism design approach to bound payoffs in a repeated game in analogous to what is done in Levin (2003). In Section IV he studies a principal-agent model in which the agent's performance is privately observed by the principal. He describes relational contracts that have the "full performance review" property, that is contracts in which the principal reports after every period (following the tradition in repeated games literature we refer to such strategies as semi-public perfect equilibria without delay). In his setup, requiring full-performance review is limiting and indeed delaying reports can improve efficiency - see footnote 22 in Levin (2003) and the discussion there, as well as Fuchs (2007). In his model value burning is always necessary to induce both effort and truthful reporting. The critical difference from our model is that on the agent's side he has moral hazard only in actions and on the principal's side only moral hazard in reports. In our model both moral hazards are on both sides of the market, which significantly complicates the analysis. A paper related to this issue is MacLeod (2003), which also studies a principal-agent problem but he has both the agent and the principal observe private signals of performance. He shows that if the signals are correlated, efficiency can be improved. That suggests that in our game one could exploit the details of the correlation in realized quantities to improve upon the lysine strategy (the case of m being known or $\underline{m}$ being bounded away from zero are extreme cases of such a correlation and we have discussed how that can be explored).

To finish this section, we show how the optimal static mechanism for a given x can be implemented as a semi-public perfect equilibrium of an infinitely repeated game if δ is high enough and $m = 2$ is the most likely outcome. Define

$$v \equiv \left(\frac{2(1-\rho_0) - \rho_1}{4\xi'(0)} \right) + (\rho_2 - \rho_0)\, x, \;\; v^N \equiv \left(\frac{2(1-\rho_0) - \rho_1}{4\xi'(0)} \right)$$

as the per period expected payoff for the optimal mechanism and the stage Nash equilibrium, respectively. When both firms report zero sales, each firm is supposed to incur a penalty of x. As the foregone value from going to the stage game Nash equilibrium is $\left(\frac{\delta}{1-\delta}\right)\left(v - v^N\right)$, we then want to realize that penalty with a probability such that the expected foregone value equals $t\,(0,0)$. Hence, when $(r_1, r_2) = (0,0)$, the equilibrium shifts to the stage game Nash equilibrium forever with probability α_0 which satisfies:

$$x = \alpha_0 \left(\frac{\delta}{1-\delta}\right)\left(v - v^N\right) \Leftrightarrow \alpha_0 = \frac{1-\delta}{\delta\left(\rho_2 - \rho_0\right)}.$$

If $(r_1, r_2) = (1,0)$ then firm 1 is to pay x and firm 2 has a zero transfer. To implement it, assume firm 1 transfers $x/2$ to firm 2 and the probability the equilibrium shifts to stage game Nash forever is α_1 which satisfies:

$$\frac{x}{2} = \alpha_1 \left(\frac{\delta}{1-\delta}\right)\left(v - v^N\right) \Leftrightarrow \alpha_1 = \frac{1-\delta}{2\delta\left(\rho_2 - \rho_0\right)}.$$

Thus, firm 1 incurs a penalty of x - as it pays $x/2$ to firm 2 and incurs an expected loss of $x/2$ from possible cartel breakdown - while firm 2 experiences no net transfer as it receives $x/2$ from firm 2 but incurs an expected loss of $x/2$ from possible cartel breakdown. Finally, if $(r_1, r_2) = (2,0)$ then firm 1 simply transfers x to firm 2. This strategy profile implements the optimal mechanism and is an equilibrium iff

$$\frac{1-\delta}{\delta\left(\rho_2 - \rho_0\right)} \leq 1 \Leftrightarrow \delta \geq \frac{1}{1+\rho_2-\rho_0}.$$

Note that the smaller is $\rho_2 - \rho_0$, the more patient firms have to be.

To summarize, assume $\rho_2 > \rho_0, \rho_1$ and firms are sufficiently patient,

$$\delta \geq \frac{1}{1+\rho_2-\rho_0}.$$

Substituting the transfer function from Theorem 3 into (7), the equilibrium price is

$$\widehat{p} = c + \frac{1}{2\xi'(0)} + x.$$

The equilibrium probability of cartel breakdown is

$$\phi\left(r_1+r_2\right)=\begin{cases}\frac{1-\delta}{\delta(\rho_2-\rho_0)} & \text{if } r_1+r_2=0\\ \\ \frac{1-\delta}{2\delta(\rho_2-\rho_0)} & \text{if } r_1+r_2=1\\ \\ 0 & \text{if } r_1+r_2=2\\ \\ \frac{1-\delta}{\delta(\rho_2-\rho_0)} & \text{if } r_1+r_2>2\end{cases}$$

and inter-firm payments are:

r_1	r_2	Payment from firm 1 to firm 2
0	0	0
1	0	$x/2$
1	1	0
2	0	x

The properties of this optimal equilibrium match those of the lysine strategy profile quite closely. First, the payment scheme is linear in the number of units; a firm transfers an amount $x/2$ to the other cartel member for each unit it reports having sold. Of particular note is that payments depend only on a firm's own sales report. Second, the probability of cartel breakdown depends only on the aggregate sales report, and is linear for equilibrium values:

$$\phi\left(r_1+r_2\right)=\left[\frac{1-\delta}{2\delta\left(\rho_2-\rho_0\right)}\right]\left(2-r_1-r_2\right).$$

3.3 Proof of Theorem 3

The way in which we will proceed is to consider a less constrained problem with a strict subset of the ICC and feasibility constraints. Once the mechanism is characterized, we'll show that the remaining ICC and feasibility constraints are satisfied.

Specifically, we seek to maximize

$$\max_{|t(r_1,r_2)|\leq x} \rho_0 t(0,0) + (\rho_1/2)[t(1,0)+t(0,1)] \tag{14}$$
$$+\left(\frac{\rho_2}{4}\right)[t(2,0)+2t(1,1)+t(0,2)]$$
$$+\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right)+\left(\frac{\rho_1}{2}\right)[t(0,1)-t(1,0)]$$
$$+\left(\frac{\rho_2}{2}\right)[t(0,2)-t(2,0)]$$

subject to these constraints:

$$t(2,0)\geq t(1,0) \tag{15}$$

$$t(2,0)\geq t(0,0) \tag{16}$$

$$\eta t(1,1)+(1-\eta)t(1,0)\geq \eta t(0,1)+(1-\eta)t(0,0) \tag{17}$$

$$0\geq t(0,2)+t(2,0) \tag{18}$$

$$0\geq t(1,1) \tag{19}$$

$$0\geq t(0,1)+t(1,0) \tag{20}$$

$$0\geq t(0,0) \tag{21}$$

(15)-(17) are the ICCs ensuring that a firm does not want to under-report its sales. $\eta\equiv\eta_{1,1}(p,p)$ so that (17) is (10) when evaluated at equilibrium prices. (18)-(21) are the feasibility constraints for when aggregate sales reports do not exceed 2.

The problem is then to choose $t(0,0), t(1,0), t(0,1), t(2,0), t(1,1)$, and $t(0,2)$ to maximize (14) subject to (15)-(21). Note that (14) is increasing in $t(1,1)$ and that $t(1,1)$ enters only (17) and (19). A higher value increases the maximand and loosens (17). Hence, (19) must be binding. Optimality then requires:

$$t(1,1)=0. \tag{22}$$

Next note that (14) is increasing in $t(0,2)$ and that $t(0,2)$ enters only (18). If $x>t(0,2)$ then optimality requires (18) to bind:

$$t(2,0)+t(0,2)=0. \tag{23}$$

If $x = t(0,2)$ then, by (18), it follows that $t(2,0) = -x$. Again, $t(2,0) + t(0,2) = 0$. Optimality then requires (23).

Using (22)-(23), defining $s = t(0,2) = -t(2,0)$, and simplifying, we can re-state (14) as choosing $t(0,0), t(1,0), t(0,1)$, and s (all in $[-x,x]$) to maximize:

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + \rho_0 t(0,0) + \rho_1 t(0,1) + \rho_2 s \tag{24}$$

subject to

$$-s \geq t(1,0) \tag{25}$$

$$-s \geq t(0,0) \tag{26}$$

$$(1-\eta) t(1,0) \geq \eta t(0,1) + (1-\eta) t(0,0) \tag{27}$$

$$0 \geq t(0,1) + t(1,0) \tag{28}$$

$$0 \geq t(0,0) \tag{29}$$

Suppose (27) was not binding:

$$(1-\eta) t(1,0) > \eta t(0,1) + (1-\eta) t(0,0).$$

Even if (28) is binding, we can raise $t(0,1)$ and lower $t(1,0)$ (note that (25) will still be satisfied) so as to satisfy (28) and, because (24) is increasing in $t(0,1)$, the payoff is higher. The only caveat to the preceding argument is if $t(0,1) = x$, in which case $t(0,1)$ cannot be increased. But then, by (28), it follows that $t(1,0) = -x$. In that case, (27) takes the form:

$$\begin{aligned} (1-\eta) t(1,0) &\geq \eta t(0,1) + (1-\eta) t(0,0) \Leftrightarrow \\ -(1-\eta) x &\geq \eta x + (1-\eta) t(0,0) \Leftrightarrow \\ -\frac{x}{1-\eta} &\geq t(0,0) \end{aligned}$$

which is a contradiction since $t(0,0) \geq -x$. Hence, (27) must be binding:

$$(1-\eta) t(1,0) = \eta t(0,1) + (1-\eta) t(0,0) \Leftrightarrow$$

$$t(1,0) = t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1), \tag{30}$$

where, using Bayes Rule,

$$\eta = \frac{\rho_2 \psi(1;2)}{\rho_2 \psi(1;2) + \rho_1 \psi(1;1)} = \frac{\rho_2 (1/2)}{\rho_2 (1/2) + \rho_1 (1/2)} = \frac{\rho_2}{\rho_1 + \rho_2}.$$

Using (30) to substitute for $t(1,0)$ in (24), the problem is now to choose $t(0,0)$, $t(0,1)$, and s to maximize:

$$\left(\frac{2(1-\rho_0) - \rho_1}{4\xi'(0)}\right) + \rho_0 t(0,0) + \rho_1 t(0,1) + \rho_2 s \tag{31}$$

subject to

$$-s \geq t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1) \tag{32}$$

$$-s \geq t(0,0) \tag{33}$$

$$0 \geq t(0,0) + \left(\frac{\rho_1 + \rho_2}{\rho_1}\right) t(0,1) \tag{34}$$

$$0 \geq t(0,0) \tag{35}$$

If an optimum has $s < 0$ then, since (31) is increasing in s, it must be the case that (32) and/or (33) are binding. By (35), if (33) binds then $s \geq 0$ which is a contradiction. Hence, if $s < 0$ is optimal then it implies (32) binds, which means that

$$t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1) > 0.$$

Since then $t(0,1) > 0$, it follows that

$$t(0,0) + \left(\frac{\rho_1 + \rho_2}{\rho_1}\right) t(0,1) > 0$$

which violates (34). Therefore, it cannot be the case that $s < 0$. We conclude that an optimum must have $s \geq 0$.

Suppose $0 > t(0,1)$. Since (31) is increasing in $t(0,1)$ then one of the constraints must bind. It follows from $0 > t(0,1)$ and (35) that (34) does not bind. When $0 > t(0,1)$, (33) binds before (32) which implies (32) does not bind. Thus, neither of the constraints involving $t(0,1)$ bind which means (31) can be increased by raising $t(0,1)$. We conclude that $t(0,1) \geq 0$ at an optimum.

To summarize the properties of an optimum derived thus far:

$$\begin{aligned} t(1,0) &= t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1) \\ t(1,1) &= 0 \\ 0 &\geq t(0,0) \\ t(0,1) &\geq 0 \\ t(0,2) &= -t(2,0) = s \geq 0. \end{aligned}$$

$t(0,1) \geq 0$ implies that if (34) holds then (35) holds which makes (35) redundant; and if (32) holds then (33) holds which makes (33) redundant. The problem is then: choose s, $t(0,0)$, and $t(0,1)$ to maximize

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + \rho_0 t(0,0) + \rho_1 t(0,1) + \rho_2 s$$

subject to

$$t(0,1) - s \geq t(0,0) + \left(\frac{\rho_1+\rho_2}{\rho_1}\right) t(0,1) \tag{36}$$

$$0 \geq t(0,0) + \left(\frac{\rho_1+\rho_2}{\rho_1}\right) t(0,1) \tag{37}$$

where (32) has been rearranged. First note that it is not an optimum for $t(0,1) - s > 0$. In that case, (37) implies (36) is not binding. Since $t(0,1) > s$ implies $s < x$, s can be increased which raises the objective while continuing to satisfy the constraints. Therefore, $t(0,1) - s \leq 0$. Hence, if (36) holds then (37) holds, and, at an optimum, $s \geq t(0,1)$.

Thus, the problem is: choose s, $t(0,0)$, and $t(0,1)$ to maximize

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + \rho_0 t(0,0) + \rho_1 t(0,1) + \rho_2 s$$

subject to

$$\begin{aligned} 0 &\geq s + t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1) \\ s &\geq t(0,1) \geq 0 \\ 0 &\geq t(0,0) \end{aligned}$$

By including the constraint $s \geq t(0,1)$, we ensure that satisfaction of (36) implies (37) holds. Suppose the first constraint does not bind at the optimum. As the objective is increasing in s, it must be the case that $s = x$. Hence, the constraint becomes:

$$0 > x + t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1),$$

but this cannot hold since $t(0,0) \geq -x$ and $t(0,1) \geq 0$. We conclude that the constraint binds:

$$0 = s + t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1).$$

Therefore, the problem is: choose s, $t(0,0)$, and $t(0,1)$ to maximize

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + \rho_0 t(0,0) + \rho_1 t(0,1) + \rho_2 s \tag{38}$$

subject to

$$0 = s + t(0,0) + \left(\frac{\rho_2}{\rho_1}\right) t(0,1) \tag{39}$$

$$s \geq t(0,1) \geq 0 \tag{40}$$

$$0 \geq t(0,0) \tag{41}$$

- Assume $\rho_2 > \rho_0, \rho_1$.

Suppose $t(0,0) > -x$. Since we've shown that, at an optimum, $t(0,1) \geq 0$ then $x > s$ by (39). But the objective can be increased by raising s by $\varepsilon > 0$ (which is possible since $s < x$) and lowering $t(0,0)$ by ε. The objective goes up by $(\rho_2 - \rho_0)\varepsilon > 0$ and, in addition, (39) still holds. Therefore, $t(0,0) = -x$.

We now have that, at an optimum, $t(0,0) = -x$ and we previously showed $s, t(0,1) \geq 0$. (39) is now

$$0 = s - x + \left(\frac{\rho_2}{\rho_1}\right) t(0,1).$$

Use this condition to substitute for s in (38):

$$\begin{aligned} & \left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) - \rho_0 x + \rho_1 t(0,1) + \rho_2 \left[x - \left(\frac{\rho_2}{\rho_1}\right) t(0,1)\right] \\ = & \left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + (\rho_2 - \rho_0) x - \left(\frac{\rho_2^2 - \rho_1^2}{\rho_1}\right) t(0,1). \end{aligned}$$

Substituting for s in (40), we get

$$s \geq t(0,1) \Leftrightarrow x - \left(\frac{\rho_2}{\rho_1}\right) t(0,1) \geq t(0,1) \Leftrightarrow \left(\frac{\rho_1}{\rho_1+\rho_2}\right) x \geq t(0,1).$$

The problem is then: choose $t(0,1)$ to maximize

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + (\rho_2 - \rho_0) x - \left(\frac{\rho_2^2 - \rho_1^2}{\rho_1}\right) t(0,1) \tag{42}$$

subject to

$$\left(\frac{\rho_1}{\rho_1+\rho_2}\right) x \geq t(0,1). \tag{43}$$

Since $\rho_2 > \rho_1$ then (42) is decreasing in $t(0,1)$. By the derived condition that $t(0,1) \geq 0$, an optimum has $t(0,1) = 0$. (Also note that since $t(0,0) = -x$ and $s \leq x$, (39) would be violated if $t(0,1) < 0$.) From $t(0,0) = -x$ and $t(0,1) = 0$, it follows from (39) that $s = x$.

If $\rho_2 > \rho_0, \rho_1$ then the solution is

$$\begin{aligned} t(0,0) &= -x \\ t(0,1) &= 0, t(1,0) = -x \\ t(0,2) &= x, t(2,0) = -x \\ t(1,1) &= 0 \end{aligned}$$

and the objective takes the value:

$$\begin{aligned} & \left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + \rho_0 t(0,0) + \rho_1 t(0,1) + \rho_2 s \\ = & \left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right) + (\rho_2 - \rho_0) x \end{aligned}$$

To complete the analysis, we need to ensure that the remaining ICC and feasibility constraints are satisfied. For that purpose, we extend the transfer function to encompass sales reports that sum to more than two.

$$\begin{aligned} t(0,0) &= -x \\ t(0,1) &= 0, t(1,0) = -x, \\ t(0,2) &= x, t(2,0) = -x \\ t(1,1) &= 0 \\ t(r_1,r_2) &= -x \text{ if } r_1 + r_2 > 2 \end{aligned} \tag{44}$$

Notice that all feasibility constraints are satisfied.

Referring back to the complete set of ICCs, the ones that we still need to verify are satisfied are, for all p_1,[8]

$$\begin{aligned} & \eta_{1,1}(p_1,\widehat{p})\, t(1,1) + \left(1-\eta_{1,1}(p_1,\widehat{p})\right) t(1,0) \\ \geq\ & \eta_{1,1}(p_1,\widehat{p})\, t(2,1) + \left(1-\eta_{1,1}(p_1,\widehat{p})\right) t(2,0) \end{aligned} \tag{45}$$

$$\begin{aligned} & \eta_{1,1}(p_1,\widehat{p})\, t(1,1) + \left(1-\eta_{1,1}(p_1,\widehat{p})\right) t(1,0) \\ \geq\ & \eta_{1,1}(p_1,\widehat{p})\, t(0,1) + \left(1-\eta_{1,1}(p_1,\widehat{p})\right) t(0,0) \end{aligned} \tag{46}$$

$$\begin{aligned} & \eta_{0,2}(p_1,\widehat{p})\, t(0,2) + \eta_{0,1}(p_1,\widehat{p})\, t(0,1) \\ & + \left(1-\eta_{0,2}(p_1,\widehat{p}) - \eta_{0,1}(p_1,\widehat{p})\right) t(0,0) \\ \geq\ & \eta_{0,2}(p_1,\widehat{p})\, t(1,2) + \eta_{0,1}(p_1,\widehat{p})\, t(1,1) \\ & + \left(1-\eta_{0,2}(p_1,\widehat{p}) - \eta_{0,1}(p_1,\widehat{p})\right) t(1,0) \end{aligned} \tag{47}$$

$$\begin{aligned} & \eta_{0,2}(p_1,\widehat{p})\, t(0,2) + \eta_{0,1}(p_1,\widehat{p})\, t(0,1) \\ & + \left(1-\eta_{0,2}(p_1,\widehat{p}) - \eta_{0,1}(p_1,\widehat{p})\right) t(0,0) \\ \geq\ & \eta_{0,2}(p_1,\widehat{p})\, t(2,2) + \eta_{0,1}(p_1,\widehat{p})\, t(2,1) \\ & + \left(1-\eta_{0,2}(p_1,\widehat{p}) - \eta_{0,1}(p_1,\widehat{p})\right) t(2,0) \end{aligned} \tag{48}$$

[8] Actually, we have already verified that (46) holds for $p_1 = \widehat{p}$.

Substituting (44) into (45),

$$-\left(1-\eta_{1,1}\left(p_1,\widehat{p}\right)\right)x \geq -\eta_{1,1}\left(p_1,\widehat{p}\right)x-\left(1-\eta_{1,1}\left(p_1,\widehat{p}\right)\right)x \Leftrightarrow \eta_{1,1}\left(p_1,\widehat{p}\right)\geq 0,$$

which holds. Next consider (46):

$$-\left(1-\eta_{1,1}\left(p_1,\widehat{p}\right)\right)x \geq -\left(1-\eta_{1,1}\left(p_1,\widehat{p}\right)\right)x.$$

Next consider (47):

$$\eta_{0,2}\left(p_1,\widehat{p}\right)x-\left(1-\eta_{0,2}\left(p_1,\widehat{p}\right)-\eta_{0,1}\left(p_1,\widehat{p}\right)\right)x \geq -\eta_{0,2}\left(p_1,\widehat{p}\right)x-\left(1-\eta_{0,2}\left(p_1,\widehat{p}\right)-\eta_{0,1}\left(p_1,\widehat{p}\right)\right)x \Leftrightarrow$$

$$\eta_{0,2}\left(p_1,\widehat{p}\right)\geq -\eta_{0,2}\left(p_1,\widehat{p}\right).$$

Finally, consider (48):

$$\eta_{0,2}\left(p_1,\widehat{p}\right)x-\left(1-\eta_{0,2}\left(p_1,\widehat{p}\right)-\eta_{0,1}\left(p_1,\widehat{p}\right)\right)x \geq -x \Leftrightarrow 2\eta_{0,2}\left(p_1,\widehat{p}\right)+\eta_{0,1}\left(p_1,\widehat{p}\right)\geq 0.$$

We conclude that if $\rho_2 > \rho_0, \rho_1$ then (44) is an optimal mechanism.

- Assume $\rho_0 > \rho_1, \rho_2$.

Return to (38) with constraints (39)-(41). Suppose $t(0,0) = 0$. Since we've already shown that, at an optimum, $s, t(0,1) \geq 0$, then (39) implies $s = 0 = t(0,1)$. Let us see if there is a better solution. Thus, suppose $t(0,0) < 0$. $t(0,0) < 0$ and (39) imply $t(0,1) > 0$ and/or $s > 0$. If $t(0,1) > 0$ then (40) implies $s > 0$. Hence, at an optimum, if $t(0,0) < 0$ then $s > 0$. If (40) is not binding - specifically, if $s > t(0,1)$ - then (38) can be increased by reducing s by ε and raising $t(0,0)$ by ε; the objective goes up by $(\rho_0 - \rho_2)\varepsilon > 0$ and (39) still holds. Given then that $s = t(0,1)$, the problem is to choose $t(0,0)$ and s to maximize

$$\left(\frac{2(1-\rho_0)-\rho_1}{4\xi'(0)}\right)+\rho_0 t(0,0)+(\rho_1+\rho_2)s$$

subject to

$$0 = t(0,0)+\left(\frac{\rho_1+\rho_2}{\rho_1}\right)s.$$

Substituting this constraint into the objective, the problem is to choose $t\left(0,0\right)$ and s to maximize

$$\left(\frac{2\left(1-\rho_0\right)-\rho_1}{4\xi'\left(0\right)}\right)+\left(\rho_1+\rho_2\right)\left(\frac{\rho_1-\rho_0}{\rho_1}\right)s \tag{49}$$

subject to

$$0=t\left(0,0\right)+\left(\frac{\rho_1+\rho_2}{\rho_1}\right)s. \tag{50}$$

Since $\rho_1-\rho_0<0$ then (49) is decreasing in s. Given (50), $t\left(0,0\right)$ should be set as high as possible, which implies $t\left(0,0\right)=0$ and, therefore, $s=0$. The best solution is then:

$$t\left(0,0\right)=0,t\left(0,1\right)=0,t\left(1,0\right)=0,t\left(0,2\right)=0,t\left(2,0\right)=0,t\left(1,1\right)=0.$$

Hence, if $\rho_0>\rho_1,\rho_2$ then no collusion can be sustained.